3 (Sem–6) MAT M 4

2020

MATHEMATICS

(Major)

Paper : 6·4

(Discrete Mathematics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times7=7$

(a) Show that for any integer n, 1 divides n.

(b) If $\tau(n)$ is odd for an integer $n > 1$, then

(i) n is odd

(ii) n is even

(iii) n is a perfect square

(iv) n is a perfect square or twice a perfect square

(Choose the correct option)

(c) Give example of two integers a and b such that

$$a^2 \equiv b^2 \pmod{3} \text{ but } a \not\equiv b \pmod{3}$$

(d) State Fermat's Little Theorem (FLT_1).

(e) Find the number of positive divisors of 7056.

(f) Write the absorption laws of propositional logic.

(g) Express 1225 as a sum of two squares.

2. Answer the following questions : 2×4=8

(a) If two co-prime integers a and b are such that a/c and b/c, then show that ab/c. Is this true when a and b are not co-prime? 1+1=2

(b) Find the remainder when 2356710825 is divided by 37.

(c) Express in disjunctive normal form :

$$1 + x_2' x_1'$$

(d) If $f(n) = \prod_{d/n} g(d)$, then show that

$$g(n) = \prod_{d/n} [f(d)]^{\mu\left(\frac{n}{d}\right)}.$$

3. Answer ***any three*** questions : 5×3=15

(a) If $a, b \in Z$, then show that a positive integer 'p' is a prime if and only if

$$p / ab \Rightarrow p / a \text{ or } p / b$$

(b) If (x, y, z) is a primitive solution of $x^2 + y^2 = z^2$, then show that one of x and y is even and the other is odd.

(c) If x and y are real numbers such that

(i) $[x+y] = [x] + [y]$ and

(ii) $[-x-y]=[-x]+[-y]$, then show that one of x or y is an integer and conversely.

(d) Show that a complete DNF is identically 1.

(e) Show that if $a_1, a_2, \ldots, a_k$ form a $RRS\ (mod\, m)$ then $k=\phi(m)$.

4. Answer **either** [(a) and (b)] **or** [(c) and (d)] :

(a) If a and b are positive integers then prove that :

$$gcd\,(a,b)\times lcm\,[a,b]=ab$$ 5

(b) If eggs are taken out from a basket two, three, four, five and six at a time, there are left over respectively one, two, three, four and five eggs. If they are taken out seven at a time, there are no eggs left over. How many eggs are there in the basket? 5

(c) If p is a prime then prove that there exist no positive integers a and b such that $a^2=pb^2$. 3

(d) Let p be a prime and $n \geq 1$ be any integer.

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ is a polynomial of degree n modulo p, then show that the congruence $f(x) \equiv 0 (mod\, p)$ has at most n mutually incongruent solution modulo p. 7

5. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Show that an odd prime p can be represented as sum of two squares if and only if $p \equiv 1 (mod\, 4)$ 7

(b) If $n \geq 1$ is an integer then show that

$$\prod_{d/n} d = n^{\tau(n)/2}.$$ 3

(c) Find all positive solutions of $x^2 + y^2 = z^2$ where $0 < z < 30$. 3

(d) If f and g are two arithmetic functions, then show that the following conditions are equivalent : 7

(i) $f(n) = \sum_{d/n} g(d)$

(ii) $g(n) = \sum_{d/n} \mu(d) f\left(\frac{n}{d}\right) = \sum_{d/n} \mu\left(\frac{n}{d}\right) f(d)$

6. Answer ***either*** [(a) and (b)] ***or*** [(c) and (d)] :

(a) Define Boolean Algebra. If A is any finite set, then show that the power set $P(A)$ form a Boolean algebra. Show that there cannot exist a Boolean algebra with three elements. 1+2+2=5

(b) Determine whether the following argument is logically correct or not :

"If I study then I will not fail in discrete mathematics. If I do not play PUBG then I will study. But I failed in discrete mathematics. Therefore, I played PUBG." 5

(c) Find a switching circuit which realizes the Boolean expression : 3

$$x\left(y(z+w)+z(u+v)\right)$$

(d) Show that the collection of connectives $\{\neg, \wedge, \vee\}$ is an adequate system. Hence deduce that $\{\neg, \wedge\}$ form an adequate system of connectives. 5+2=7
