2019

MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the formula that relates three sides and two angles of a spherical triangle.

(b) State the Napier's rules to write all formulae of a right-angled spherical triangle.

(c) What are the approximate right ascension and declination of the sun on 21st March and 21st September?

(d) Write the limits between which the sum of the three angles of a spherical triangle lie.

(e) Which points on the celestial sphere are called cardinal points?

(f) Distinguish between geocentric and heliocentric conjunction of two planets.

(g) Explain briefly where must a star be situated as to have no displacement due to annual parallax.

2. Answer the following questions : $2\times4=8$

(a) Prove that altitude of celestial pole is equal to the latitude of the place of the observer.

(b) Describe with a suitable diagram, what is meant by rising and setting of a star.

(c) Prove that the sides of a polar triangle are supplements of the angles of its primitive triangle.

(d) If T is the orbital period of a planet, show that a small increase Δa in semimajor axis a will produce an increase $\frac{3T}{2a}\Delta a$ in the period.

3. Answer any *three* parts of the following : $5\times3=15$

(a) The right ascension and declination of a star are given. Explain, how you will find celestial longitude and latitude.

(b) If a is the sun's altitude in the prime vertical at a place of latitude ϕ and L is the longitude, prove that

$$\phi = \sin^{-1}(\sin\varepsilon \sin L \operatorname{cosec} a)$$

ε being the obliquity of the ecliptic.

(c) Show that the velocity of a planet in its elliptic orbit is

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

where $\mu = G(M+m)$ and a is the semimajor axis of the orbit.

(d) Prove that the apparent path of a star on account of parallax is an ellipse.

(e) If h and H be the hour angles of a star of declination δ on the prime vertical (west) and at setting respectively, for a place in north latitude, show that

$$\cos h \cos H + \tan^2 \delta = 0$$

4. *(a)* State and prove the cotangent formula related to a spherical triangle. 1+5=6

(b) In a spherical triangle ABC, prove that

$$\frac{\sin(A+B)}{\sin C} = \frac{\cos a + \cos b}{\cos c}$$ 4

5. *(a)* Derive Cassini's formula for refraction in the form

$$\sin R = \frac{a \sin \zeta}{a+h}\sqrt{\mu^2 - 2\mu\cos R + 1}$$

stating the assumption used. a, h, ζ have their usual meanings. 6

Or

Examine the effect of refraction on the times of rising and setting of the sun.

(b) What do you mean by sidereal day and mean solar day? For a star, prove that

$$\text{sidereal time} = \alpha + H$$

where α is the right ascension and H is the hour angle of the star. 2+2=4

6. *(a)* Define lunar eclipse. With a neat diagram, discuss the commencement of lunar eclipses of different types. 5

(b) Show that the angle subtended at the earth's centre by the centre of the sun and the moon at the beginning of solar eclipse is

$$D = r_{\odot} + r_{c} + P' - P$$

where $r_{\odot}$ and r_{c} are the angular radii of the sun and the moon respectively and P, P' are their parallaxes. 5

Or

Define geocentric parallax. What is meant by horizontal parallax? Discuss the effects of geocentric parallax on the right ascension and declination of a star. 1+2+7=10

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