2019

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the moment of inertia of a solid sphere of radius a and mass M about a diameter.

(b) Define equimomental systems.

(c) State the theorem of parallel axes on moment of inertia.

(d) Define the centre of oscillation of a compound pendulum.

(e) What is the principle of conservation of energy?

(f) A particle moving freely in space requires three coordinates (x, y, z), to specify its position. What is the degree of freedom of the particle?

(g) What are generalized coordinates?

2. Answer the following questions : 2×4=8

(a) A particle of mass 4 units is placed at the point $(-1, -1, 1)$. What is the product of inertia of the particle about $OX - OY$; and $OY - OZ$?

(b) A particle of mass 3 units is located at the point $(2, 0, 0)$. The particle rotates about O with angular velocity $\vec{\omega} = \hat{k}$. Find the angular momentum of the particle about O.

(c) A rigid body with one point fixed rotates with angular velocity $\vec{\omega}$ and has angular momentum $\vec{\Omega}$. Prove that the kinetic energy is given by

$$T = \frac{1}{2}(\vec{\omega} \cdot \vec{\Omega})$$

(d) A particle of mass m moves in a conservative force field. Write the Lagrangian function.

3. Answer the following questions : 5×3=15

(a) Show that the moment of inertia of a rectangular lamina of mass M and sides $2a$, $2b$ about a diagonal is

$$\frac{2M}{3} \frac{a^2 b^2}{a^2 + b^2}$$

Or

Find the product of inertia of a semi-circular wire about diameter and tangent at its extremity.

(b) State and prove d'Alembert's principle.

(c) Show that the momental ellipsoid at the centre of an elliptic plate is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\left(\frac{1}{a^2}+\frac{1}{b^2}\right)z^2 = \text{constant}$$

Or

Show that a uniform rod of mass M is kinetically equivalent to three particles, rigidly connected and situated one at each end of the rod and its middle point, the masses of the particles being $\frac{1}{6}M$, $\frac{1}{6}M$ and $\frac{2}{3}M$.

4. A rod of length $2a$, is suspended by a string of length l, attached to one end, if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, show that

$$\frac{3P}{a}=\frac{(4\tan\theta-3\tan\phi)\sin\phi}{(\tan\phi-\tan\theta)\sin\theta}$$ 10

Or

(a) A rough uniform board of mass m and length $2a$, rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Find the distance through which the board moves in this time. 6

(b) A circular board is placed on a smooth horizontal plane and a body runs round the edge of it at a uniform rate. What is the motion of the board? 4

5. (a) Prove that the time of complete oscillation of a compound pendulum is

$$2\pi\sqrt{\frac{k^2}{gh}}$$

where k is the radius of gyration of the body about a fixed axis and h is the distance of centre of inertia of the body from the fixed axis. 5

(b) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion. 5

6. A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding. Discuss the motion. 10

Or

Obtain the equation of motion of a rigid body under impulsive forces. 10

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