2019

MATHEMATICS

(Major)

Paper : 1.1

(Algebra and Trigonometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Does the set of all integers form a group with respect to addition of integers?

(b) What is the degree of the following permutation?

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$$

(c) Write Lagrange's theorem (Group theory).

(d) Write generators of the multiplicative cyclic group

$$\{1, -1, i, -i\}$$

(e) Find the argument of the complex number $-1+i\sqrt{3}$.

(f) Express $\cosh y$ in the power of e^{y} and e^{-y}.

(g) Use Gregory's series, find the value of

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\infty$$

(h) Define symmetric matrix.

(i) What is the normal form of a matrix?

(j) What is the echelon form of the matrix $A=\begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}$?

2. Answer the following questions : $2\times5=10$

(a) Give with an example that union of two subgroups of a group is not necessarily a subgroup of the group.

(b) Express the following matrix as a sum of symmetric and skew-symmetric matrices :

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$$

(c) If

$$A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} \text{ and } B = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$$

then show that B is the inverse of A.

(d) If A is an $n \times n$ non-singular matrix, then prove that

$$|\text{adj } A| = |A|^{n-1}$$

(e) Write $-i$ in the form

$$r(\cos\theta + i\sin\theta)$$

3. Answer the following questions : 5×2=10

(a) Prove that every subgroup of a cyclic group is cyclic.

(b) Express $\log_e(x+iy)$ in the form $A+iB$, where $x, y \in R$.

Or

Prove that $\alpha^2+\beta^2 = e^{-(4n+1)\pi\beta}$ if $i^{\alpha+i\beta} = \alpha + i\beta$.

4. Answer any *two* questions of the following : 5×2=10

(a) If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, then find the value of $\Sigma\alpha^3$ in terms of p, q and r.

(b) If the roots of the equation

$$x^3 - ax^2 + bx - c = 0$$

be in harmonic progression, then show that the mean root is $\frac{3c}{b}$.

(c) Apply Descartes' rule of signs to find the nature of the roots of the equation $3x^4 + 12x^2 + 5x - 4 = 0$.

5. Answer any *one* part : 10

(a) Let A be a non-empty set and let R be an equivalence relation in A. Let a and b be arbitrary elements in A. Then prove that—

(i) $[a] = [b]$ iff $(a, b) \in R$;

(ii) either $[a] = [b]$ or $[a] \cap [b] = \phi$.

(b) State and prove fundamental theorem on equivalence relation.

6. Answer any *one* part : 10

(a) *(i)* If H is any subgroup of G and $h \in H$, then prove that $Hh = H = hH$.

(ii) If a, b are any two elements of a group G and H any subgroup of G, then prove that $Ha = Hb \Leftrightarrow ab^{-1} \in H$.

(b) If H is a subgroup of G, then prove that there is a one-to-one correspondence between the set of left cosets of H in G and the set of right cosets of H in G.

7. Answer any *one* part : 10

(a) Separate into real and imaginary parts of $(\alpha + i\beta)^{x+iy}$.

(b) If $\sin(\alpha + i\beta) = x + iy$, then prove that—

(i) $x^2 \operatorname{cosec}^2 \alpha - y^2 \sec^2 \alpha = 1$;

(ii) $x^2 \operatorname{sech}^2 \beta + y^2 \operatorname{cosech}^2 \beta = 1$.

8. Answer any *one* part : 10

(a) For what values of η, the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \eta$$
$$x + 4y + 10z = \eta^2$$

have a solution? Solve them completely in each case.

(b) Prove that every square matrix A can be expressed in one and only one way as $P + iQ$, where P and Q are Hermitian matrices.

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